A Neutral Suggestion

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In the formalisms we attempt to create for program calculation a humble but important role is played by what has been variously called the *unit*, *identity element* or just *identity* of a (dyadic) operation. Richard Bird has used the notation $id_\oplus$ in several papers, and Roland Backhouse has introduced the notation $1_\oplus$ for this in his unpublished Exploration paper. Recently I noticed that Richard has taken to writing $0_\oplus$.

Many of the most important operations like $+\!\!\!+$ and $\uplus$ are more of an additive than of a multiplicative nature, and it is indeed perhaps a bit confusing to the innocent student if $1_+ \;=\; 0$. On the other hand, if the operation *is* multiplicative, like $\times$ itself, and has a zero element, then $0_\times \;=\; 1$, as Richard's new convention would have it, is equally confusing.

An identity element is also called a *neutral* element. This suggest the notation $\nu_\oplus$, from the Greek letter *nu*, the first letter of the word "neutral" (which is of Greek origin). It has the advantage that it is neutral with respect to the perceived nature of the operation. Then we have:

$$
\begin{aligned}
\nu_{+\!\!\!+} &= [\,] \\
\nu_{\uplus} &= \Lbag\,\Rbag \\
\nu_{\circ} &= id \\
\nu_{+} &= 0 \\
\nu_{\times} &= 1 \\
\nu_{\mathsf{X}_\odot} &= \Lbag \nu_\odot \Rbag
\end{aligned}
$$

What about zeros (absorbing elements) of a dyadic operation? Turning to Greek we have the adjective *anabrotic*, which means "gobbling up" (derived from *bibrōskein*, to devour). The word is listed in Webster as meaning "corrosive". Unfortunately, unlike "neutral", this word is virtually unknown. Still, it shares its first letter with the common word "absorbing", and so we could use:

$$
\begin{aligned}
\alpha_{\times} &= 0 \\
\alpha_{\wedge} &= \mathsf{false} \\
\alpha_{\mathsf{X}_\odot} &= \nu_{\uplus}
\end{aligned}
$$